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Design-Based Research for Developing and Refining Differential Calculus Instructional Lesson (DCIL)

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Abstract

Malaysian students often struggle with the abstract and complex concepts of calculus, primarily due to a weak foundational understanding of functions, which is a fundamental element in learning calculus. Traditional teaching methods in Malaysia further make worse this issue by emphasizing computational procedures over conceptual understanding. Addressing these challenges, this study aims to design and refine a Differential Calculus Instructional Lesson to enhance students' understanding. Using a design-based research methodology, the study followed three phases: preparation and design, teaching experiment, and retrospective analysis with intervention refinement. In the first phase, a diagnostic test identified students' specific difficulties related to graphs and graphing in differential calculus. A teaching experiment was then conducted in a pre-university class, where data were gathered through classroom observations, students' work on the Desmos platform, and interviews with a teaching witness. Analysis revealed that the instructional lesson effectively provided visual aids that aided learning, but most students only attained a basic, Action-level understanding of graphs and graphing. The retrospective analysis highlighted key areas for improvement. The lesson's extensive content within a two-hour session limited deep learning. To address this, two refinements were introduced, which are introducing a draft box feature for students to comfortably draft their answers and a narrowed focus on linear and quadratic graphs and their derivatives. These adjustments aimed to align the lesson with students' cognitive load and time constraints. This study contributes valuable insights into the application of design-based research for developing effective educational interventions. The refinement phase demonstrated the need to align instructional content with students' learning capacities. Findings offer practical guidance for educators and instructional designers on integrating theoretical frameworks into design-based research cycle and teaching strategies, ultimately address learning gaps and promote conceptual understanding in differential calculus.

Keywords: Action-Process-Object-Schemas (APOS) Theory, Design-Based Research, Differential Calculus, Hypothetical Learning Trajectories, Merrill's First Principle of Instruction

Introduction

Differentiation topic is one of the fundamental branches of calculus, which is an essential subject in the fields of Science, Technology, Engineering, and Mathematics (STEM). Differentiation primarily deals with rates of change, slopes of functions, graphing, and

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its application in real-world contexts, making it a crucial component in mathematical modelling (Saad, Abu Mansor, Azudin, & Mohd Hamdi, 2024). In Malaysia, calculus concepts have been introduced since secondary school as part of the additional mathematics syllabus (Awang Salleh & Zakaria, 2012; Awang & Zakaria, 2013). According to Parrot and Leong (2018), Malaysian students often struggle to understand the abstract and complex ideas inherent in calculus, leading to frustration and a lack of understanding. One major contributing factor is their weak foundation in understanding functions, which is a fundamental concept in calculus (Drlik, 2015).

Traditional calculus classes often emphasize computational procedures without placing enough importance on conceptual understanding. These traditional teaching strategies tend to present a "list of procedures to follow," leading students to just practice the routine algebraic manipulations without truly understanding the underlying concepts. According to Mendezabal and Tindowen (2018), the traditional approach prevents students from effectively applying calculus concepts, as it emphasizes algebraic techniques rather than adopting a balanced approach that includes graphical, numerical, and analytical methods. In the current teaching and learning environment for calculus, there is a challenge in achieving a balance between conceptual understanding and procedural fluency. This imbalance can limit students' understanding of calculus concepts and their ability to apply them to real-world problem situations (Awang Salleh & Zakaria, 2012).

According to Harris (1996), our current education system focuses too much on giving students extensive practice with exercises and assuming that repeated practice leads to mastery of the topic. While this approach may improve procedural skills, it prevents students from achieving a deeper level of mathematical understanding. As a result, students may find it challenging to apply the concepts they have learned to real-world problems due to a lack of conceptual depth. Therefore, an integrated approach is needed to enhance both conceptual understanding and procedural fluency which will improve students’ mathematical competency.

Visualisation is one of the key elements in solving mathematical problems especially in word problems (Ahmad et al., 2010). Ahmad Tarmizi et al. (2010) and Makgakga and Makwakwa (2016) found that students often struggle with understanding the concepts of limits, derivatives, and integrals, which are fundamental to calculus. These difficulties arise because students are unable to visualise the mathematical concepts, especially those related to function transformations which are the essential aspects of calculus (Fitriani, Pasaribu, Novitasari, Samosir, & Yusmiati, 2023). Makgakga and Makwakwa (2016) stated that students' struggles with mastering calculus concepts lead to poor learning outcomes and difficulties in problem-solving. Therefore, incorporating technology into the teaching and learning of calculus is crucial as it can help students visualise these concepts. Numerous studies have explored the use of technology in teaching mathematics, including the use of graphic calculators (Chien, 2019; Parrot & Leong, 2018), GeoGebra (Listiana, Aklimawati, Wulandari, & Isfayani, 2022; Sari, Hadiyan, & Antari, 2018), and Geometer’s Sketchpad (Ganesan & Leong, 2020; Kotu & Weldeyesus, 2022).

When incorporating technology into the calculus learning process, it should not be assumed that learning will automatically improve (Vajravelu & Muhs, 2016) and only look for the improvement on the students’ result. Instead, there should be a focus on the correct and appropriate implementation of technology tools and a better understanding of how students construct their understanding when exposed to newly designed instructional lessons. Innovation occurs when it suits students' needs, and the technology

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is implemented thoughtfully in the curriculum. Current implementations of technology or instructional design often perform well in organizing and delivering courses. However, there is a lack of attention to the implementation process and students' learning experiences, particularly their knowledge construction in these learning environments (Badali, Hatami, Farrokhnia, & Noroozi, 2020; Cai & Moallem, 2022), and students' active engagements and interactions (Yılmaz, Unal, & Cakir, 2017).

Therefore, this study is conducted to design an instructional lesson namely Differential Calculus Instructional Lesson (DCIL) which is incorporating with the use of Desmos. The teaching and learning process of differential calculus, including instructional notes and group activities, fully utilised Desmos. Students' understanding is explored using APOS theory when instructors use this newly designed instructional lesson.

This study integrates APOS Theory, Hypothetical Learning Trajectories (HLT), and Merrill's First Principles of Instruction to provide a comprehensive framework for designing instructional lesson aided with Desmos and also for the exploration of students' understanding of differential calculus. APOS Theory is essential for analysing students' cognitive processes involved in concept formation, ensuring that students' progress from action-based understanding to higher-order thinking (Borji, Alamolhodaei et al., 2018). HLT provides a structured learning pathway, allowing for systematic instructional design that aligns with students' cognitive development (Simon, 2014). Merrill's First Principles further enhance the framework by emphasizing active learning through the activation of prior knowledge and guided application, ensuring that students not only develop conceptual understanding but also apply their knowledge effectively (Truong, Elen, & Clarebout, 2019).

The need for this integrated approach stems from gaps in existing research, where instructional methods often focus on either cognitive development or instructional strategies in isolation. Maqsood, Ceravolo, Ahmad, and Sarfraz (2023) highlighted that traditional methods frequently fail to fully leverage learning trajectories in instructional design, opting instead for data mining and visualization techniques to analyse students' course trajectories. Additionally, traditional approaches often lack a structured mechanism for supporting and understanding students' cognitive development, leading to fragmented learning experiences. By integrating APOS Theory for cognitive structuring, HLT for instructional scaffolding, and Merrill's Principles for learner-centered instructional design, this study provides a more systematic and effective instructional model aimed at improving student learning outcomes in differential calculus.

This study is guided by the following research questions:

1. How does students' mathematical understanding of graphs and graphing in differential calculus develop based on APOS theory when they engage with a Desmos-based instructional lesson?
2. To what extent does students' actual learning align with the hypothetical learning trajectory as identified through retrospective analysis?
3. What refinements can be implemented to improve the design of the Differential Calculus Instructional Lesson (DCIL)?

Literature Review

APOS Theory in Learning Calculus

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The Action-Process-Object-Schema (APOS) theory is a constructivist framework extensively employed to investigate students' understanding of mathematical concepts, particularly in calculus. Founded by Dubinsky (1991), it is valued for its ability to bridge research and teaching (Oktac, Trigueros, & Romo, 2019). According to Dubinsky (1991), APOS theory posits that learning mathematical concepts involves constructing mental structures: actions, processes, objects, and schemas. These structures are essential for students to achieve a profound understanding of mathematical concepts.

In the context of calculus, APOS theory has been utilised to analyse students' understanding of concepts such as limits, derivatives, and integrals. Researchers have developed genetic decompositions, which describe the mental constructions necessary for students to fully understand these concepts (Cetin, 2009; Maharaj, 2010). Studies employing APOS theory reveal that many students struggle to progress from an action or process conception of calculus concepts to a more robust understanding involving objects and schemas (Borji & Martínez-Planell, 2020; Listiawati & Juniati, 2021; Nagle, Martínez-Planell, & Moore-Russo, 2019). For instance, students may be proficient in computing derivatives procedurally but find it challenging to interpret derivatives graphically or conceptually (Borji, Font, et al., 2018).

Borji, Alamolhodaei et al. (2018) implemented the APOS-ACE (Action, Process, Object, Schema - Activities, Classroom discussion, Exercises) instructional approach to help students develop appropriate mental structures. This approach involves designing activities, classroom discussions, and exercises to guide students through constructing actions, processes, objects, and schemas for differential calculus concepts. The following outlines the key aspects of each APOS level that facilitate students' understanding of calculus:

(a) Action- Initially, students understand derivatives through procedural actions, such as calculating derivatives using formulas or rules, and they require external guidance. This action-based understanding is crucial for developing a deeper comprehension of derivatives, particularly in choosing the correct formulas or rules for calculations (Maharaj, 2013).

(b) Process - As students advance, they begin to perceive derivatives as processes and can perform derivative calculations without explicitly executing them. They start to conceptualise the derivative process mentally (Borji, Alamolhodaei, et al., 2018).

(c) Object - Subsequently, students develop an object-based understanding of derivatives, allowing them to visualise and interpret the derivative as a mathematical entity. This stage involves encapsulating the process understanding into an object, including comprehending the derivative as a slope, a rate of change, or an instantaneous rate of change (Maharaj, 2010).

(d) Schema - Finally, students construct a schema for derivatives, a mental framework integrating actions, processes, and objects. This schema enables students to apply derivatives in various contexts and recognize the derivative as a fundamental concept in calculus (Borji, Font, et al., 2018; Maharaj, 2010).

APOS theory proves beneficial in guiding and assisting students in developing a comprehensive understanding of calculus, transitioning from procedural actions to deeper conceptual insights (Borji & Martínez-Planell, 2020; Listiawati & Juniati, 2021; Nagle et al., 2019). Thus, this study employs APOS theory to explore students' understanding of differential calculus when exposed to newly designed instructional lessons.

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Hypothetical Learning Trajectories in Instructional Lesson Design

Hypothetical Learning Trajectory (HLT) is the theoretical model utilised by researchers, teachers, and curriculum developers to design mathematics instruction aimed at conceptual learning. HLTs consist of three key components: a learning goal, a set of learning tasks, and a hypothesized learning process. This model was first proposed by Simon (1995). HLTs are crucial for discovering and understanding how students learn and develop their comprehension of complex concepts. They are designed to guide students through the learning process, identifying key landmarks and potential obstacles as they transition from a naive to a more sophisticated understanding of a concept (Ivars, Fernández, Llinares, & Choy, 2018). The development of HLTs involves understanding students' current knowledge, describing fundamental aspects of assimilation and fixation, selecting appropriate tasks, and preparing teachers for possible interventions (Morales Carballo, Damián Mojica, & Marmolejo Vega, 2022). Table 1 presents an example of the Hypothetical Learning Trajectory used in this study.

Table 1: Example of Hypothetical Learning Trajectory and Actual Learning Trajectory Table with APOS Genetic Decomposition

Hypothetical Learning Trajectory			Actual Learning	
Task Formulation	Conjecture of how students would respond	Evaluation for students' understanding based on APOS theory	Transcript Excerpt/ Clarification	Quantitative impression of how well the conjecture and actual learning matched (e.g., −, 0, +)

Source: Adapted from Bakker & van Eerde, 2015, p.442

Merrill's First Principles of Instruction

Based on Merrill's (2002) first principles of instruction, instructional activities should be centered on real-world problems or tasks, such as optimization functions or graphing and modelling population growth. According to Merrill, the design of instructional lessons consists of four phases. First, educators should activate students' prior knowledge. In this phase, educators engage learners by helping them recall and stimulate their existing knowledge, connecting it with new information (Merrill, 2018). For instance, they might use derivative concepts to model the motion of objects or optimise business incomes. Second, educators should use clear examples to demonstrate new knowledge and illustrate the desired learning outcomes (Jalilehvand, 2016).

Next, students should apply their new knowledge in authentic contexts (Choi, 2014). They integrate their knowledge into problem-solving activities, such as solving optimisation problems using calculus by incorporating these principles, educators can create engaging and effective instructional lessons for students in differential calculus, thereby promoting deeper understanding and transferable knowledge.

There were few studies have explored the use of the first principles of instruction in instructional design. For example, Badali et al. (2020) examined their application in MOOC design to explore student learning and satisfaction. Their study found that incorporating these principles led to improved student learning outcomes and higher satisfaction levels. The structured approach helped students engage more effectively with the course content. Cai and Moallem (2022) focused on redesigning an online graduate course using a rapid prototype approach whereas Cheung and Hew (2015) explored their application in designing a blended learning course. Gardner (2011) investigated improving student performance in introductory biology courses. These findings suggest that the First Principles of Instruction serve as a valuable framework

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for designing effective learning experiences that promote meaningful and lasting learning.

Desmos as an Instructional Tool for Teaching and Learning

Desmos is a graphing calculator available as both a web-based application and mobile platforms. It has several interactive features such as Activities Builder, Graphing Calculator, and Classroom Activities, where educators can use to create engaging classroom activities to attract their students in the lessons, or assessment tasks to evaluate their students' mathematical understanding. The use of Desmos is not limited to mathematics or science teachers in their classroom teaching but also can be used by language teachers to develop task-based games to engage their students in designed learning environment that can help in improving their communication skills (Caniglia, Borgerding, & Meadows, 2017). The activities designed aims to encourage students to be active listeners, with those in the position to give the instruction able to deliver clear communication.

Unlike traditional hand-held technology tools such as graphic calculator, Desmos is more intuitive and simpler to operate (Ebert, 2015). Normally, altering graph dimensions on graphing calculator involves numerous steps, however, it is much simpler on Desmos, where we can just simply press the plus/minus button on the screen to zoom in and out the graph. Such simplicity directly addresses a common issue associated with the use of technology, which is the intrinsic complexity of the technology tools that prevents students from efficiently using it (Hillman, 2014). This is supported by Thomas' (2016) argument that students spent too much time on the mechanics of operating the typical graphic calculator, which the simplicity of Desmos may alleviate the potential stress among students using it when completing math problems. By reducing technological frictions, it creates a classroom environment where learning is both accessible and intellectually stimulating.

Educators are encouraged to employ a number of innovative tools to demonstrate various forms of mathematical relationships such as in graphical, symbolic, or tabular form. Technology can aid in the understanding of mathematical concepts if implemented appropriately (Gertenbach & Bos, 2016). For example, Ebert (2015) employed Desmos as a platform to assign a graphing project to his students to reinforce their comprehension of function concepts. The study demonstrated how students' ability to graph the general shapes of functions, graph transformation, and graphing functions based on the restricted domain and range significantly enhanced after completing the project task. Besides that, Desmos is a platform for teachers to evaluate their students' mathematical understanding on the concepts learned (Gulati, 2016; Zheng, Naresh, & Edwards, 2020). Students can articulate their ideas and problem-solving processes on the assigned activities board where teachers can view them in real time. This function is useful especially for teachers who wish to directly track their students' progress in understanding of the lesson content.

Methodology

This study employed a design-based research (DBR) method to design an instructional unit namely Differential Calculus Instructional Lesson (DCIL). The main aim is to explore students' understanding on differential calculus. DBR is chosen for this study because it enables the iterative development, analysis and refinement of the designed instructional lesson (Kennedy-Clark, 2013), making it particularly suitable for educational settings where controlled experiments may not be feasible. Besides that,

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DBR bridges the gap between theory and practice by addressing real educational challenges while simultaneously generating theoretical insights (Barab & Squire, 2004).

There are three important phases in this DBR approaches which are preparation and design phase, teaching experiment phase, and lastly, retrospective analysis and refinement phase. DBR emphasizes on iterative cycles of design, implementation during teaching experiment phase, and continuous refinement of instructional interventions based on real-world feedback and outcomes. The scope of this study was limited to the first cycle of the design-based research process to ensure a detailed yet concise discussion, as including multiple cycles would result in an excessively lengthy report.

Preparation and Design Phase

A diagnostic test was given to 32 students who participated in this research during the preparation and design phase. Their scores on the diagnostic test served as an initial benchmark of their knowledge of differential calculus. Additionally, the test analysis result obtained helped researchers understand which calculus concepts needed emphasis and to be included in the intervention during the design process (Gravemeijer, 1994). The test content was validated by three mathematics experts, each having over 10 years of teaching experience in the mathematics subject to ensure accuracy and relevance. The analysis of the test results was then used to develop a hypothetical learning trajectory (HLT) for the lesson. The genetic decomposition of students' mental constructions which based on APOS theory (Dubinsky & McDonald, 2001) in relation to their understanding of each designed activity was included into the HLT as well. The lesson was designed in order to address the difficulty areas that identified from the test analysis, and it aimed to provide targeted instruction to improve students' understanding of differential calculus. Table 2 presented the students' APOS Achievement for the Diagnostic Test.

Table 2: Students' APOS Achievement for Diagnostic Test

Question	Percentage of Students (%)				
	Action	Process	Object	Schema	Zero Marks/ No answer given
1	25%	34.3%	18.8%	6.3%	15.6%
2	6.3%	40.6%	6.3%	21.8%	25%
3	21.9%	28.1%	28.1%	0%	21.9%
4	50%	15.6%	0%	0%	34.4%
5	40.7%	12.5%	0%	3.1%	43.7%

Source: Author’s work

The nature of Question 1 required students to apply the quotient rule to find the derivative of a given function and identify the x-coordinates of stationary points. Question 2 tested students' understanding of the gradients of tangent and normal lines, as well as their interrelationship. Question 3 involved an optimisation scenario where students had to determine the maximum volume of water that could be held in a water tank. Question 4 tasked students with interpreting information from displayed graphs, identifying their slopes, and subsequently sketching the graphs of their derivatives. Question 5 instructed students to graphically represent a cubic function based on a provided equation.

Analysis of Table 2 revealed weaknesses among students in graph interpretation and graph-related problems, specifically with few achieving Object and Schema levels. A significant percentage of students either scored zero marks or did not attempt Questions 4 and 5. Consequently, the researcher opted to emphasize topics related to

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graphs and graphing in differential calculus. Table 3 below presents the Hypothetical Learning Trajectory used in the DCIL instructional lesson, incorporating the Desmos platform as part of this study.

Table 3: Hypothetical Learning Trajectory with APOS Genetic Decomposition

Hypothetical Learning Trajectory (HLT)			
Activity	Merrill's First Principles of Instruction	Conjecture of how the lesson be conducted	Students' understanding evaluation based on APOS theory
Introduction	Activation	The instructor helps the students recall their knowledge of the definition of the derivative of a function.	NA
Lesson Content	Demonstration	The instructor demonstrates several graphs, including linear, quadratic, and cubic graphs, on the Desmos platform. The instructor guides the students in determining the increasing and decreasing intervals for the displayed graphs. The instructor guides the students in relating the positive and negative values of the derivative function to the increasing and decreasing intervals of the functions.	Action: The students are able to determine the position of the point where the graph changed its slope. Process: The students are able to determine the increasing and decreasing intervals from the stationary points. Object: The students are able to relate the increasing and decreasing behavior of the graph f to the positive and negative values of the derivative graph on the corresponding intervals. Schema: The students are able to establish relationships between the derivative function and the original function, regardless of the type of function.
Classroom Discussion	Application and Integration	Students are organized into groups to engage in the Matching Graph Cards activity on the Desmos platform. Instructor monitors their strategies and progress using the Teacher Dashboard within the Desmos platform.	Action: Students capable of identifying pairs of linear and quadratic graphs to form original-derivative pairs. Incorrect responses may occur as they recall that the derivative of a function is one degree lower than the original graph. Process: Through mental repetition of differentiation techniques, students successfully match a horizontal line as the derivative of a linear graph, reflecting the constant nature of the derivative of a linear function. Object: Students establish connections between intervals of increase or decrease in the original function's graph and the positive or negative values of its derivative over corresponding intervals.

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			Schema: By relating graph properties such as slope, tangent lines, original functions, derivatives, and derivatives at critical points, students construct a comprehensive framework for accurately matching derivative functions with their respective original function graphs.
Take Home Exercise	Application and Integration	Instructor briefs and shares the Desmos link with students, enabling them to complete the task independently at home.	NA

Source: Authors' work

Teaching Experiment Phase

Following the completion of the instructional lesson design and its implementation, a teaching experiment was conducted in a real pre-university classroom setting. This research was carried out in a private institution in Sarawak with a sample of 32 students. For the sampling technique, a purposive sampling approach was employed. The selection of participants was determined by the institution, focusing on a group of science pre-university students who were the target learners for the instructional intervention. In this experiment, the researcher assumed the role of the instructor, diverging from traditional experimental designs where researchers typically do not serve as instructors (Cobb, Confrey, Disessa, Lehrer, & Schauble, 2003; Molina, Castro, & Castro, 2007). The researcher collaborated closely with a teacher familiar with the classroom environment (Collins, Joseph, & Bielaczyc, 2004), who acted as a teaching witness throughout the experiment. This teacher provided critical feedback and advice on the implementation process, highlighting areas for improvement or modification. The teaching witness facilitated real-time reflection and refinement of the instructional lesson, ensuring a strong connection between students' learning and their experience with the new intervention (Cobb et al., 2003; Hoadley, 2004).

The two hours lesson adhered to Merrill's First Principles of Instruction (Merrill, 2002) and the flow outlined in the Hypothetical Learning Trajectory (HLT) in Table 3, incorporating Desmos-assisted activities. Students' understanding was evaluated using the APOS theory (Asiala et al., 1997). Data collection during this phase included video recordings of the lesson, student work on the Desmos platform, and interview with the teaching witness. The interview was crucial for gaining deeper insights into participants' thought processes (Charters, 2003), enabling the researcher to better understand the teaching witness's professional opinion and feedback on the newly designed DCIL instructional lesson, particularly regarding students' mental construction of the mathematical concepts. Based on this feedback, the instructional lesson was modified or refined for subsequent cycles of the teaching experiment.

Retrospective Analysis and Intervention Refinement Phase

The retrospective analysis and intervention refinement are critical elements in the third phase of design-based research. The retrospective analysis provided real-time insights into how students learned and constructed their knowledge during the lesson. It allowed the researcher to critically review the teaching experiment process by comparing students' actual learning trajectories with the hypothetical learning trajectories,

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assessing how well they matched. This analysis was based on video recordings of the lesson, which were transcribed into text. Information about students' mental constructions and their engagement with Desmos activities was recorded and entered into the appropriate sections of Table 1 as per displayed in Section "Hypothetical Learning Trajectories in Instructional Lesson Design".

Validity and Reliability of this Study

To ensure validity and reliability in this study, several strategies were implemented. Content validity was ensured through expert validation, where three mathematics experts reviewed the diagnostic test to confirm its alignment with differential calculus concepts and its effectiveness in identifying students' difficulties. Additionally, the instructional content was designed based on these identified challenges and reviewed for its relevance. Construct validity was maintained by grounding the study in APOS theory, which provided a structured framework for assessing students' understanding levels. The use of a hypothetical learning trajectories (HLT) table as a benchmark further strengthened the validity of the evaluation.

Triangulation was employed by collecting multiple sources of data, including interviews with the teaching witness, classroom video recordings, and students' work on Desmos enhance the validity. This approach provided a more comprehensive and accurate analysis. Additionally, observer triangulation was incorporated by involving the course coordinator as the teaching witness of the lesson in observing lessons and students' responses able to reduce potential bias in data interpretation.

For reliability**,** the study ensured inter-rater reliability by having teaching witness independently observe the real-classroom situations and students' Desmos work. A structured coding scheme based on APOS theory in hypothetical learning trajectory table was used to minimize subjective interpretation.

By integrating these validity and reliability measures, the study ensured a rigorous and trustworthy evaluation of students' understanding of differential calculus using Desmos and DBR methodology**.**

Findings

This study obtained three main data sources, which were video recordings from classroom observations during teaching experiment phase, students' work on the Desmos platform, and interview with the teaching witness.

Classroom Observation and Students' Work on Desmos

Figure 1 presented an overview of the classroom conditions and seating arrangement. Students were seated in rows in a computer lab, with a large projector screen positioned at the centre of the room. One camera was placed in the middle of the classroom to capture the overall environment, while four additional cameras were positioned at the front of each row to record group interactions.

At the beginning of the lesson, the instructor initiated a discussion to reflect on and revise students' prior knowledge regarding the definition of derivatives. This activity was designed to activate students' previous knowledge. Students were engaged in typing their answers on the Desmos platform, while the instructor monitored their responses via the teacher dashboard to gauge their understanding. It was noted that students were primarily focused on typing and had minimal interaction with their peers. After a short

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while, the instructor displayed students' answers on the screen and facilitated a discussion about their understanding of the definition of derivatives.

Figure 1: Overview of Teaching Experiment Classroom Situation
Source: Authors' work

Then the instructor initiated a discussion about graphically understanding the properties of the original and derivative graphs. She engaged the students by asking them a few questions on determining the increasing and decreasing intervals of a function. Initially, students showed confusion regarding the intervals and points, having difficulty distinguishing between the two. This was evident when a student questioned if the word "interval" in the question referred to the distance between points. The instructor then tried to clarify by pinpointing the regions as intervals and a specific dot as a point using Desmos.

Instructor : Do you understand what is the difference between interval and points? Anyone?
Student 2 : Is interval about the distance between points?
Instructor : Yes, intervals represent distances between points.
(ClassroomObservation_L1)

Initially, almost 50% of the students demonstrated a basic understanding of graph properties, i.e. at the Action level with many unable to state the increasing interval correctly. One student even asked the instructor what "increase" meant. Most of these students did not use accurate x-coordinates when identifying increasing or decreasing regions on the Desmos platform, often mistakenly identifying the x-intercept as the starting point of these regions. The instructor further explained the concept to help students correct their misunderstandings based on the displayed graphs. Figure 2 showed examples of two students mistakenly used x-intercept as starting point of increasing region.

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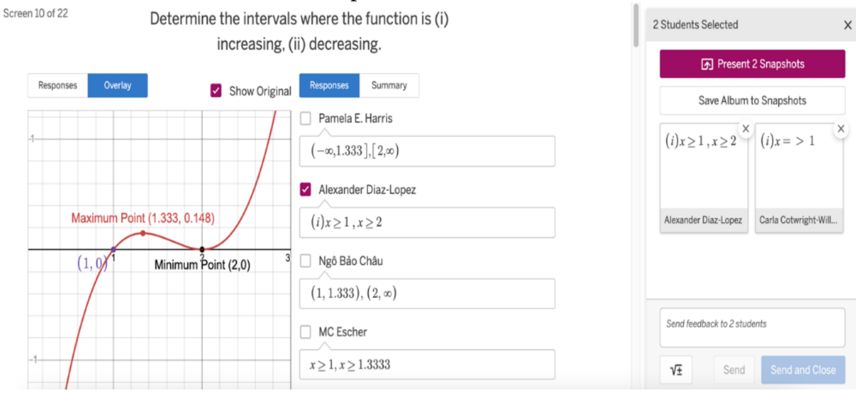

Figure 2: Example of Students' Misconception on determine the Starting Point of Increasing Interval
Source: Authors' work

During the group activity on Matching Graph Cards in the Desmos platform, varying levels of participation were observed. Some students actively participated, while others remained seated and worked alone until the instructor encouraged them to cooperate with others. From the students' work on Desmos as per displayed on Figure 3, it was evident that students had limited understanding of matching functions with their derivatives as there was a few responses involved matching the horizontal graph with other horizontal graph and linear graphs with another linear graph. This showed that these students were in Action level of understanding of the derivative concept. Towards the end of the lesson, it was clear that the instructor was running out of time to discuss this group activity further, which posed challenges in covering all the necessary lesson content.

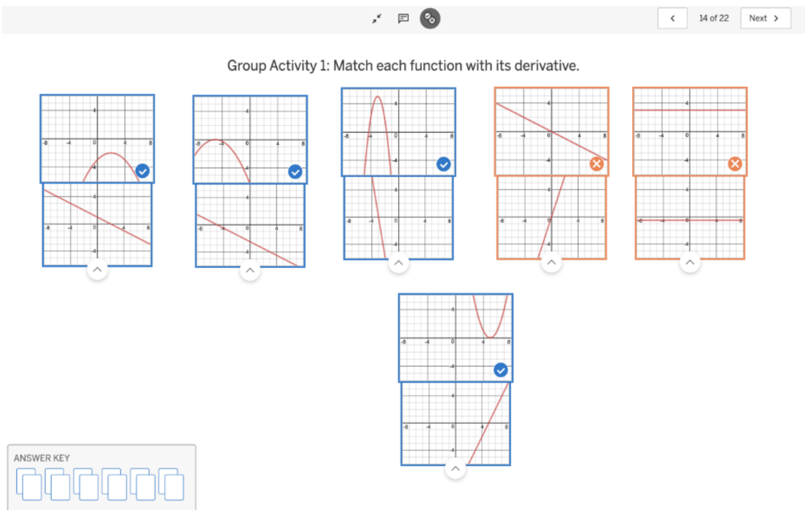

Figure 3: Example of Students' Misconception Matching Graphs Activity
Source: Authors' work

Interviews with Teaching Witness
The interviews with the teaching witness revealed three main themes, which are strengths of the designed instructional lesson, weaknesses of the lesson, and suggestions for improvement. The teaching witness observed that the Differential Calculus

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Instructional Lesson (DCIL) effectively delivers lesson content through a practical approach where it is allowing students to visualize and manipulate graphs on the Desmos platform. This approach differs from traditional methods, where students only can listen verbal explanations from the instructor and must imagine the graphs.

Teaching Witness: And the thing is a live thing because they were able to draw, to sketch, and they were also able to move the mouse here and there to see how the graph looks like, and so on.
Teaching Witness: Rather than just using graph paper or pencils to draw or to imagine how the graph looks like.
(Interview_TW: 10 & 14)

The integration of group discussions and student presentations through the Desmos platform was noted by the teaching witness as a key element in engaging students. Displaying student work on the screen and discussing various solutions increased interaction between the instructor and students.

Teaching Witness: I also found out that it was very interactive because this app promoted engagement between the instructor and the student, as well as among the students in the group.
(Interview_TW: 8)

The second theme identified was the weakness of the DCIL lesson. The teaching witness expressed concerns about excessive content and two hours is insufficient for students to process and understand the lesson fully. The lack of time for exploring more examples or practices was also highlighted.

Teaching Witness: But I notice that, perhaps it is too much for the students to understand all of it in just one lesson.
Teaching Witness: Um... the content is too much for them within the two-hour lessons.
(Interview _TW: 16&18)

From the observation by teaching witness, another difficulty encountered by students was adapting to graph-displayed questions instead of the usual equation-based questions. The teaching witness observed that students were shocked when first introduced to these types of questions.

Teaching Witness: The students were shocked when first saw the questions. We seldom ask them to differentiate it by just looking at the graph.
Teaching Witness: I mean the function without equation given but just the graph.
(Interview_TW:58& 62)

The third theme involved suggestions from the teaching witness for refining the instructional lesson to better suit students' needs. One suggestion was to have only one person, such as a group leader in submitting answers for group activities on the Desmos platform. This is to allow other students to focus on discussions.

Researcher : Is there any suggestion for improvement for this lesson?
Teaching Witness: Okay, for this lesson, regarding the group activity, I think it's better if only the leader of a group can access and submit the answers.
(Interview_TW: 72-73)

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Another suggestion was to enhance the use of Desmos for better teaching practices, such as incorporating an online whiteboard feature or a draft box for side-by-side calculations. The teaching witness praised the platform's ability to share student answers anonymously through the teacher dashboard.

Teaching Witness: Erm… I'm not sure whether it can be done or not. Maybe there are other features that we can add to Desmos, such as a blank whiteboard or draft box.
Researcher : Okay…
Teaching Witness: We can arrange it in a side-by-side view, with what you are currently teaching on one side and another view for the whiteboard or the blank screen.

(Interview _TW: 84-88)

Teaching Witness: I observed that after you finished any exercises, the students can share their solution on the instructor's screen. Everyone can also refer to it. I believed that if we continue using this app in the teaching and learning, it will be able to minimize the time taken by the instructor to explain.

(Interview _TW: 98-99)

Retrospective Analysis and Refinement
Table 4 presented the retrospective analysis comparing the Hypothetical Learning Trajectory (HLT) with the Actual Learning Trajectory (ALT).

Table 4: Retrospective Analysis Outcomes

Task formulation	Hypothetical Learning Trajectory Conjecture of how the lesson be conducted
Introduction	1. The instructor helps the students recall their knowledge about the definition of the derivative of a function.
Lesson Content	1. The instructor demonstrates several graphs, including linear, quadratic, and cubic graphs, on the Desmos platform. 2. The instructor guides students in determining the increasing and decreasing intervals for the displayed graphs. 3. The instructor helps students relate the positive/negative values of the derivative function to the increasing/decreasing intervals of the functions.
Classroom Discussion	1. Students are divided into groups to discuss the Matching Graph Cards activity on the Desmos platform. 2. The instructor monitors their strategies and progress through the Teacher Dashboard on the Desmos platform.
Take-home Exercises	The instructor gives the Desmos link to the students to complete the task at home.

Source: Authors' work

Several important refinements were made to this lesson after compiling and analysing the data obtained from classroom observations, student work, feedback from teaching witnesses through interviews, and retrospective analysis of the HLTs and ALTs. One significant change was the reduction of the lesson content from covering all three types of functions to focusing on just two types, which are linear and quadratic functions with their derivatives. This adjustment was made because observations and interviews with

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teaching witnesses revealed that the original content was too extensive for a two-hour lesson, making it difficult for students to process and understand the material effectively.

By emphasizing these two types of functions, it is believed that students will be able to focus better and learn the important properties of the graphs of linear and quadratic functions and their derivatives. Once students have a solid understanding of these concepts, they can then apply what they have learned to cubic functions and their derivatives.

Additionally, based on a recommendation from a teaching witness, a draft box was added to the slides of the Desmos instructional lesson. The draft box allows students to draft their answers without submitting them to the teacher dashboard for sharing with their classmates. This feature helps students prepare their solutions more comfortably and allows the instructor to review only the completed final solutions. Figure 4 illustrates the overview of the slides before refinement, while Figure 5 shows the interface of the slide after amendments to the DCIL lesson.

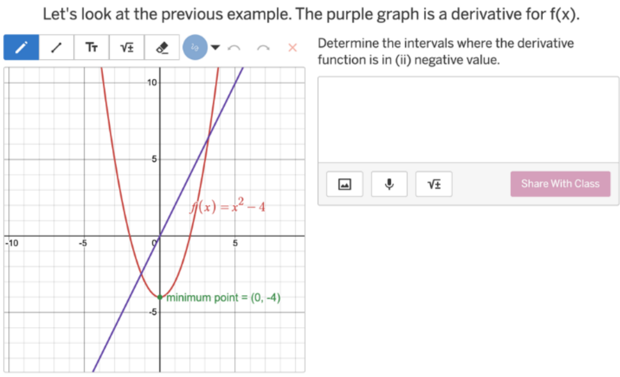

Figure 4: The Interface of the Slide before Refinement
Source: Authors' work

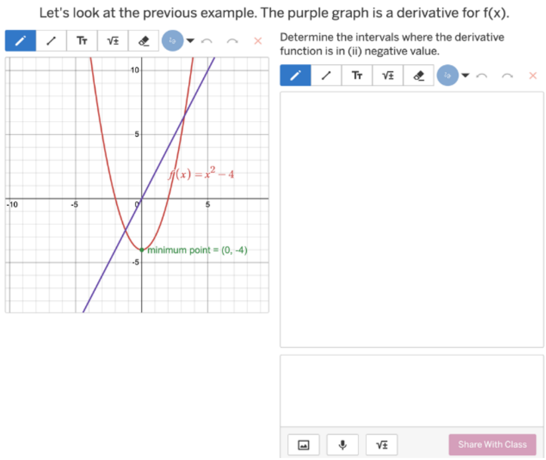

Figure 5: The Interface of the Slide after Refinement
Source: Authors' work

Discussion

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From the diagnostic test results, it was observed that less than 4% of students were able to achieve the Object and Schema levels of understanding in Questions 4 and 5, which involved graphing problems and applications of differential calculus. This finding was reinforced through classroom observations and analysis of students' work on the Desmos platform. These observations revealed that most students demonstrated proficiency only at the Process level of understanding. This indicates a significant initial deficiency in students' understanding of differential calculus concepts. Similar trends have been reported in previous studies by Nagle et al. (2019), and Maharaj and Ntuli (2018) who also found that most participants operated predominantly at the Process level when engaging with calculus concepts.

Interviews with the teaching witness highlighted the effectiveness of the Differential Calculus Instructional Lesson (DCIL) in allowing students to visualise and manipulate graphs on the Desmos platform. This method contrasts with traditional approaches, where students rely solely on verbal explanations and are required to mentally imagine graphs. The practical hands-on approach aligns with Yimer's (2022) findings which demonstrated that integrating cooperative learning with GeoGebra enabled students to better visualise abstract concepts through technology. Furthermore, Rolf and Slocum (2021) emphasized that effective instructional lessons depend on strong interaction between instructors and students, a criterion the DCIL successfully fulfilled according to the teaching witness. Another strength of the DCIL is its iterative and flexible design process, which supports continuous refinement and adaptation to address students' learning needs. This reflects key characteristics of design-based research as outlined by Bakker and van Eerde (2015) and Wittmann (2019), emphasizing the importance of ongoing development, implementation, and evaluation of instructional materials and their impact on students' learning processes.

Despite its strengths, the teaching witness identified several areas for improvement in the DCIL. One notable challenge was the inability to complete all tasks within the two-hour lesson timeframe. Keiser and Lambdin (1996) observed that innovations in mathematics teaching often require more time due to the incorporation of diverse strategies and assessment approaches. Similarly, the Organisation for Economic Co-operation and Development (2020) warned that attempting to cover both the depth and breadth of content within constrained timeframes can lead to content overload. To address this, it is essential to plan carefully when designing new interventions in teaching and learning. Another issue noted was that students struggled with graph-based questions when equations were not explicitly provided. Habre and Abboud (2006) mentioned that traditional approaches focusing solely on memorising formulas and symbolic aspects hinder students' understanding and visualisation of graph properties. This lack of familiarity hindered students' initial adaptation to the DCIL instructional lesson.

One recommendation from the teaching witness on suggestions for refining the instructional lesson was to implement task distribution among group members. Assigning specific roles within groups encourages better engagement in discussions and collaborative work. This approach is supported by Bénéteau, Guadarrama, Guerra, Lenz, Lewis, and Straumanis (2017), who found that assigning roles fosters communication skills and increases engagement through structured practice. Another critical suggestion was the need for educators to develop a deeper understanding of Desmos' functionalities. Research by Shé, Bhaird, et al. (2023) and Shé, Fhloinn, et al. (2023) highlighted the importance of integrating technology into academic environments to engage students

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effectively. Future instructional designs could benefit from incorporating more advanced functionalities of platforms like Desmos, further enhancing their effectiveness in teaching complex mathematical concepts.

Limitation/Implications/Conclusion

This study aimed to design DCIL, an instructional lesson with activities facilitated by Desmos Classroom Activities to explore pre-university students' understanding of differential calculus concepts. To achieve this, a methodology integrating Asiala's APOS theoretical analysis with Merrill's First Principles of Instruction and a hypothetical learning trajectory based on design-based research was used to design DCIL. The refinement was finalised after a teaching experiment and based on the data analysis obtained to create a better instructional lesson that suits the students' needs and their mental constructions. This study has significant benefits for curriculum developers, pre-university lecturers or instructors, and students. It provides valuable insights for curriculum developers and instructors on how to incorporate technology such as Desmos into differential calculus curriculum and lessons. It offers guidelines for implementing design-based instructional strategies in mathematics lessons. Most importantly, this study provides the greatest benefit for students, allowing them to experience different learning strategies where DCIL offers visualisation for learning differential calculus and engages them in a more interactive learning environment. By addressing its current limitations and leveraging technology effectively, the instructional design using design-based research approach can better support students in achieving higher levels of understanding in in challenging mathematics topics like differential calculus.

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